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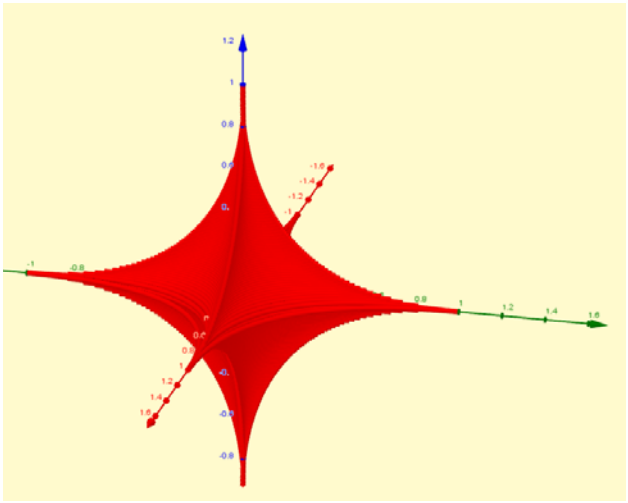
Fórmula de Dirichlet

$$R=\left\{(x,y,z)\in\mathbb{R}^3|\left(\frac{x}{a}\right)^{\alpha}+\left(\frac{y}{b}\right)^{\beta}+\left(\frac{z}{c}\right)^{\alpha}\leq 1;x,y,z\geq 0\right\}$$

$$I_R=\iiint_R x^{p-1}y^{q-1}z^{r-1}dx\,dy\,dz=\frac{a^p b^q c^r}{\alpha\beta\gamma}\frac{\Gamma\left(\frac{p}{\alpha}\right)\Gamma\left(\frac{q}{\beta}\right)\Gamma\left(\frac{r}{\gamma}\right)}{\Gamma\left(\frac{p}{\alpha}+\frac{q}{\beta}+\frac{r}{\gamma}+1\right)},\quad \Gamma(p)=(p-1)!$$

Ejemplo:



$$R=\left\{(x,y,z)\in\mathbb{R}^3|\sqrt{|x|}+\sqrt{|y|}+\sqrt{|z|}\leq a\right\}$$

$$\alpha=\beta=\gamma=\frac{1}{2},\qquad a=b=c,\qquad p=q=r=1$$

$$V_R=8\frac{a^3}{\left(\frac{1}{2}\right)^3}\frac{\left(\Gamma(2)\right)^3}{\Gamma(3\cdot 2+1)}=\frac{4}{45}a^3$$

Tomado de: Pedro Puig Adam, *Calculo Integral*. Lección 24, nota § 10 (pg. 225)

Se generaliza a cualquier número de dimensiones, incluida 2, de forma obvia.